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Buckling and post-buckling analysis for magneto-elastic–plastic ferromagnetic beam-plates with unmovable simple supports

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Abstract

This paper presents an analysis of buckling/snapping, bending and post-buckling/snapping behaviors of magneto-elastic–plastic interaction and coupling for soft ferromagnetic beam-plates with geometrically nonlinear deformation and unmovable simple supports at the ends of the plates. Based on the expression of magnetic force from the variational principle of ferromagnetic plates, the theory of thin plates with the nonlinear deformation of van Karman's type, and the Mises yield criterion and the increment theory for plastic deformation, here, we establish a numerical code to quantitatively simulate the behaviors of the nonlinearly multi-coupling problems by the finite element method. Along with that the phenomena of buckling, bending, and post-buckling/snapping, or the characteristic curve of deflection versus magnitude of applied magnetic fields are numerically displayed, the critical values of buckling/snapping and yield magnetic fields, and the expansibility of plastic region after the plates undergo plastic deformation with increasing of the applied magnetic fields, as well as the evolvement of deflection configuration of the plate are numerically obtained in a case study.

Keywords: Ferromagnetic beam-plates; Magneto-elastic–plastic interaction; Geometric nonlinearity; Critical buckling/snapping and yield magnetic fields; Numerical analysis

1. Introduction

In the past four decades, the electromagnetic materials and structures are extensively employed in high-technique, e.g., fusion reactor, microelectronics, magnetic levitation, etc., as intelligent materials or element structures. Following it, much more researches were carried out to find the characteristic of electro-magneto–mechanical interaction. When applied electromagnetic fields are strong enough to the structures,

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it is necessary to consider those behaviors of mechanics, such as buckling, post-buckling, intensity, dynamics, etc., of the structures in the design of safety and function.

Moon and Pao (1968) were the first to conduct the experiment of magneto-elastic buckling of cantilevered ferromagnetic beam-plates in a transverse magnetic field, and gave an analytic explanation for it. Following their work, many researchers paid their attention on the experimental and theoretical studies of magneto-elastic interaction of ferromagnetic structures in a magnetic field (Pao and Yeh, 1973; Popelar, 1972; Miya et al., 1978, 1980; Eringen, 1989; Zhou et al., 1995; Zhou and Zheng, 1999b; Zhou, 2000; etc., for example). In order to commonly describe those two distinct experimental phenomena, i.e., the bucking/instability (Moon and Pao, 1968) and the increasing of frequency of free vibration (Tagaki et al., 1995), Zhou and Zheng (1997, 1999a) proposed a generalized variational principle to get expressions magnetic forces for the soft ferromagnetic plates and bodies, along with that they gave some qualitative and quantitative analyses of the experimental phenomena (Zhou et al., 1995; Zhou and Miya, 1998). In addition, Zhou et al. (1995) demonstrated that the buckling phenomenon occurs only when the applied magnetic field is in the transverse direction to the flat-plate, while the plate generates bending deflection when the plate is subjected to an oblique magnetic field. From the theory of plates, we know that the *buckling* phenomenon of plates implies that the bending deflection is generated only when the magnitude of an applied source is larger than a critical value. Then the deflection increases rapidly even when the applied source has a little of increasing over the critical value, which is referred to as the well-known *post-buckling* phenomenon. If a plate undergoes a deflection no matter how small the applied source is, the model is called as *bending*. As the applied source increases, the bending deflection becomes larger till a *snapping* phenomenon, sometimes, occurs. For the case of post-buckling/snapping, it is usual that deflection is large enough such that to pursue the deflection path theoretically, the geometrical nonlinear relation of thin plates should be taken into account.

Recently, Zhou et al. (2000) gave a numerical analysis of buckling and post-buckling to the ferromagnetic plates with geometrically nonlinear elastic deformation under applied magnetic fields. Zheng and Wang (2001) studied similar problem to the ferromagnetic rectangular plates with nonlinear magnetization. It should note that almost all researches in the magneto-mechanical interaction were conducted for the elastic deformation of ferromagnetic structures no matter how a magnetic field is strong so as to lead to high intensity of stress possibly. It is obvious that the deformation of structures increases rapidly after they are in the path of post-buckling and post-snapping through bending. In this case, there is a possibility that the structures undergo plastic deformation when some stress is high over the yield point of the material used. As the knowledge of authors, few research except for Littlefield (1996) has been found in literature to report the magneto-mechanical coupling and interaction of ferromagnetic structures in affiliation with plasticity, since the multi-coupling and multi-nonlinear problem of ferromagnetic structures with elastic–plastic deformation in strong magnetic fields is much more complex and difficult to get an analysis of the behaviors. For the situation of research to instability of elastic–plastic plates made of normal materials, here, we have noted the work of Wu and Yu (1986), Liu et al. (1989), Müller et al. (1993), and Tvergaard (1999) as examples.

In this paper, we will generalize the study into the magneto-elastic–plastic interaction and coupling for ferromagnetic plate structures in strong magnetic fields in order to find the effect of plastic deformation on the phenomena, especially, the critical magnetic values of buckling, snapping, and plastic yield which are directly related to the safety of structures operated in strong magnetic fields. For this purpose, here, we establish a numerical code to the analysis. In the following section, some essential equations and formulae employed are briefly introduced. Following them, the methodology for the numerical code is displayed in Section 3, while some numerical results to a case study of the unmovably simply supported beam-plate made of soft ferromagnetic material in transverse and oblique magnetic fields are shown in Section 4. Finally, some remarks and conclusions are given in Section 5.

2. Basic equations

In this section, we briefly introduce some basic equations for the problem considered here. As shown in Fig. 1, we consider a soft ferromagnetic beam-plate of unit width, length L, and thickness h in an applied magnetic field of uniform distribution of magnitude B_0. When $\alpha \neq 0$, we refer to the plate as being in an oblique magnetic field. Otherwise, it is called as in a transverse magnetic field when $\alpha = 0$. The elastic–plastic relation of the material is taken as shown in Fig. 1(b), to which we denote the elastic constant of Young's modulus by Y, the linear-strain-hardening with the coefficient of hardening by H', and the yield stress by σ_s. After that, according to the theory of plasticity (Owen and Hinton, 1980), the value of the ultimate moment in the fully plastic condition to a thin beam-plate with rectangular cross-section can be calculated in terms of the yield stress σ_s, i.e., $M_p = \sigma_s(bh^2/4)$.

2.1. Increment theory of plasticity

Here, we only quote those equations what we will employ in the analyses of the problem considered in this article. From the theory of plasticity (Owen and Hinton, 1980; Kachanov, 1971), we know that the increment of strain, $\mathrm{d}\varepsilon_{ij}$, can be divided into two parts, elastic and plastic. That is

$$\mathrm{d}\varepsilon_{ij} = (\mathrm{d}\varepsilon_{ij})_e + (\mathrm{d}\varepsilon_{ij})_p \tag{1}$$

where the subscripts "e" and "p", respectively, imply elasticity and plasticity; "i" and "j" are the index corresponding to the Cartesian coordinate axes in the representation of Einstein's rule of index. The elastic increment of strain $(\mathrm{d}\varepsilon_{ij})_e$ can be formulated by (Kachanov, 1971)

$$(\mathrm{d}\varepsilon_{ij})_e = \frac{\mathrm{d}\sigma'_{ij}}{2\mu} + \frac{(1-2v)}{3Y}\delta_{ij}\mathrm{d}\sigma_{kk} \tag{2}$$

in which μ and v are the elastic constants of shearing modulus and Poisson's ratio, respectively; δ_{ij} is the Kronecker δ-function; $\mathrm{d}\sigma_{lm}$ $(l,m = 1,2,3)$ and $\mathrm{d}\sigma'_{ij}$ $(i,j = 1,2,3)$ are, respectively, the increments of stress and deviatoric stress components. When the subscript indexes become same, e.g., $l = m$, it implies summation of the quantity with respect to the repeated subscript indexes from 1 to 3 according to Einstein's rule

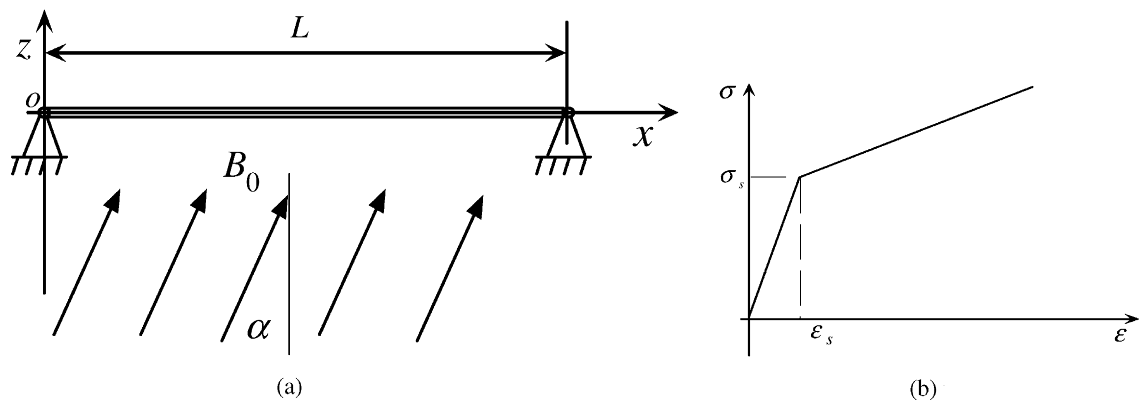

Fig. 1. Schematic drawing of a soft ferromagnetic beam type plate in a uniform oblique magnetic field ($\alpha \neq 0$). When $\alpha = 0$, we call the magnetic field as a transverse magnetic field. (a) Ferromagnetic beam-plate with unmovable simple supports at two ends in uniformly distributed oblique magnetic fields; (b) the stress–strain relation of elastic–plastic material employed.

of index. For the plastic increment of strain, it is known that this is in proportional to the gradient of the plastic potential Q with respect to stress, i.e.,

$$(\mathrm{d}\varepsilon_{ij})_{\mathrm{p}} = \mathrm{d}\lambda \frac{\partial Q}{\partial \sigma_{ij}} \tag{3}$$

in which $\mathrm{d}\lambda$ is the ratio factor, referred to as the plastic multiplier. In general case, the plastic potential Q is chosen by a plastic yield function f such that Eq. (3) can be rewritten as

$$(\mathrm{d}\varepsilon_{ij})_{\mathrm{p}} = \mathrm{d}\lambda \frac{\partial f}{\partial \sigma_{ij}} \tag{4}$$

For the case that the Mises yield criteria/function is employed here, we may get

$$\frac{\partial f}{\partial \sigma_{ij}} = \sigma'_{ij} \tag{5}$$

Further, Eq. (4) can be reduced into

$$(\mathrm{d}\varepsilon_{ij})_{\mathrm{p}} = \mathrm{d}\lambda \sigma'_{ij} \tag{6}$$

Here, σ'_{ij} represents the deviatoric stress. Eq. (6) is attributed to the Prandtl–Reuss equation in the theory of plasticity, to which a plastic methodology based on it is referred to as the flow theory of plasticity (Owen and Hinton, 1980).

Substitution of Eqs. (2) and (4) into Eq. (1) leads to the strain–stress relation of the elastic–plastic problem of the matrix form

$$\mathrm{d}\varepsilon = [D]^{-1}\mathrm{d}\sigma + \mathrm{d}\lambda \frac{\partial f}{\partial \sigma} \tag{7}$$

in which $[D]$ is the matrix of elastic constants; the superscripts "-1" indicate the inverse of the matrix. According to the flow theory of plasticity, we know that the strain–stress relations of Eq. (7) can be further expressed as $\mathrm{d}\sigma = [D]_{\mathrm{ep}}\mathrm{d}\varepsilon$ where $[D]_{\mathrm{ep}}$ is a matrix of elastic–plastic constants.

2.2. Bending equations of thin beam-plate

Here, for simplicity, we consider a beam type plate, i.e., a rectangular thin plate with unit width, made of soft ferromagnetic material with the unmovable simple supports at two ends, and with the nonlinear deformation of van Karman's type. Further, we assume that the plate consists of the parts of full plastic elements and full elastic elements as doing in Owen and Hinton (1980). For this case, the differential equation for bending of the plate may, similar to Xu (1990) for geometrical nonlinear beam-plate of elasticity, after those quantities of the flexural rigidity and the axial internal force are replaced by ones of that the plastic deformation is considered, be formulated by

$$D^* \frac{\mathrm{d}^4 w}{\mathrm{d}x^4} - N_x^* \frac{\mathrm{d}^2 w}{\mathrm{d}x^2} = q_z^{\mathrm{em}}(x), \quad 0 < x < L \tag{8}$$

$$w(x)|_{x=0} = 0, \quad \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=0} = 0 \tag{9}$$

$$w(x)|_{x=l} = 0, \quad \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=l} = 0 \tag{10}$$

Here, $w = w(x)$ is the deflection function of the plate while x stands for the coordinate along the longitudinal direction of the plate; D^* indicates the flexural rigidity of the plate; N_x^* is the internal extension/compress force along the longitudinal direction. For the case of bending deformation of a thin plate undergoing the elastic–plastic deformation with strain–stress relation shown in Fig. 1(b), one can obtain the deflection rigidity of the form

$$D^* = \begin{cases} D_0, & M < M_\mathrm{p} \\ D_0\left(1 - \dfrac{D_0}{D_0 + H'}\right), & M > M_\mathrm{p} \end{cases} \tag{11}$$

in which $D_0 = Yh^3/[12(1-v^2)]$ is the flexural rigidity of the plate at where of cross-section only elastic deflection generates. For the internal force N_x^*, considering the condition of unmovable supports at two ends and the constitutive relation of plasticity employed here as well as the geometrical nonlinearity of von Karman's type, we get

$$N_x^* = \frac{6}{h^2 L}\left(\int_0^L D^*\left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}x\right) + \frac{H'h}{(D_0 + H')L}\sigma_\mathrm{s} l_\mathrm{p} \tag{12}$$

Here, l_p means the length of total plastic elements.

2.3. Equations for magnetic fields

For the static state of a soft ferromagnetic plate in an applied magnetic field, we can introduce a scalar magnetic potential ϕ which is related to the magnetic field vector $\mathbf{H}$ by $\mathbf{H} = -\nabla\phi$ when electric charge, electric field and electric current are not taken into account in this research. After that, the Maxwell equations of electromagnetic fields for linear electromagnetic medium can be reduced into the form (Zhou et al., 1995)

$$\nabla^2\phi^+ = 0 \quad \text{in } \Omega^+(w) \tag{13}$$

$$\nabla^2\phi^- = 0 \quad \text{in } \Omega^-(w) \tag{14}$$

with the connection equations

$$\phi^+ = \phi^- \quad \text{on } S(w) \tag{15}$$

$$\mu_\mathrm{r}\frac{\partial\phi^+}{\partial n} = \frac{\partial\phi^-}{\partial n} \quad \text{on } S(w) \tag{16}$$

as well as the boundary condition

$$-\nabla\phi^- = \frac{1}{\mu_0}B_0 \quad \text{at } \infty \tag{17}$$

Here, $\Omega^+(w)$ and $\Omega^-(w)$, respectively, represent the inside and outside regions of the deflected ferromagnetic plate; μ_0 and μ_r are the magnetic permeability of vacuum and the relative magnetic permeability of ferromagnetic medium, respectively; $S(w)$ denotes the enclosed surface of $\Omega^+(w)$; $\mathbf{n}$ stands for the unit normal vector outward to the surface $S(w)$. It should be noted that the regions $\Omega^+(w)$ and $\Omega^-(w)$, and the surface $S(w)$ all are dependent on the deflection $w = w(x)$.

2.4. Expression of magnetic force

Due to that the transverse deflection is prominent in the displacements of thin plates, here, we only consider the application of transverse magnetic force exerted on the middle plane of plates. From Zhou and Zheng (1997, 1999b), we take the equivalent transverse magnetic force as follows:

$$q_z^{\mathrm{em}}(x) = \frac{\mu_0\mu_{\mathrm{r}}\chi}{2}\{(H_n^+(x,h/2))^2 - (H_n^+(x,-h/2))^2\} - \frac{\mu_0\chi}{2}\{(H_\tau^+(x,h/2))^2 - (H_\tau^+(x,-h/2))^2\} \tag{18}$$

which can commonly describe those two distinct experimental phenomena, i.e., the buckling of ferromagnetic plate in transverse magnetic fields (Moon and Pao, 1968; Zhou et al., 1995) and the increasing of frequency of free vibration of the plate in in-plane magnetic fields (Tagaki et al., 1995; Zhou and Miya, 1998). Here H_n^+ and H_τ^+ are the normal and tangential components of the magnetic field vector $\mathbf{H}$ on the top ($z = h/2$) and the bottom ($z = -h/2$) surfaces of the plate, respectively; and χ ($= \mu_{\mathrm{r}} - 1$) stands for the magnetic susceptibility.

From the above equations, one can find that the mechanical deformation of the plate structure is nonlinearly coupled with the magnetic fields, while the plate is nonlinear both geometrically and materially.

3. Numerical approach for magneto-elastic–plastic interaction

In order to quantitatively analyze the magneto-mechanical phenomena of the ferromagnetic plates with the elastic–plastic deformation when the plates are subjected to strong magnetic fields, we use the increment finite element method (IFEM) and an iterative approach to realize it.

3.1. Methodology of IFEM for elastic–plastic deformation

Dividing the beam-plate along longitudinal direction to the plate into N elements for the deflection of plate, we get a system of nonlinear algebraic equations with the matrix form (Owen and Hinton, 1980):

$$[K_T]_{\mathrm{ep}}\{w\} = \{R\} \tag{19}$$

where $\{R\}$ is the column with elements of equivalent load at nodes of element and is related to the magnetic force of Eq. (18); $\{w\}$ stands for the column of nodal deflection at the middle plane of plate; $[K_T]_{\mathrm{ep}}$ represents the global rigidity matrix which consists of the elements of flexural rigidity coefficients with either elastic or plastic deformation, or both. Due to the geometric and material nonlinearity considered, it is evident that we have the following dependence to the global rigidity matrix:

$$[K_T]_{\mathrm{ep}} = [K_T(\{w\},\{\sigma\})]_{\mathrm{ep}} \tag{20}$$

which means that the elements of the matrix are dependent upon both the deflection $\{w\}$ and the stress $\{\sigma\}$ which are constrained by the elastic–plastic relation introduced above. In order to efficiently solve the strong nonlinear simultaneous algebraic equations of Eq. (19), here, we take the increment approach of IFEM, that is

$$[K_T(\{w\}_1^{(j-1)},\{\sigma\}_1^{(j-1)})]\Delta\{w\}_1^{(j)} = \Delta\{R\} \tag{21}$$

in which $[K_T(\{w\}_1^{(j-1)},\{\sigma\}_1^{(j-1)})] = [K_T(\{w\},\{\sigma\})]|_{\{w\}=\{w\}_1^{(j-1)},\{\sigma\}=\{\sigma\}_1^{(j-1)}}$ where $\{w\}_1^{(j-1)} = \{w\}_0 + \Delta\{w\}_1^{(j-1)}$ and $\{\sigma\}_1^{(j-1)} = \{\sigma\}_0 + \Delta\{\sigma\}_1^{(j-1)}$ are, respectively, the iterating values of $\{w\}_1$ and $\{\sigma\}_1$ corresponding to the load that a pre-specified increment load $\Delta\{R\}_1$ is added. Here, the initial iterating values of $\{w\}_1$ and $\{\sigma\}_1$ are, respectively, taken as $\{w\}_1^{(0)} = \{w\}_0$ and $\{\sigma\}_1^{(0)} = \{\sigma\}_0$, where $\{w\}_0$ and $\{\sigma\}_0$ are the solutions of the nonlinear equation (19) before the increment load $\Delta\{R\}_1$ is added, or $\{R\} = \{R\}_0$. Thus, we successively obtain the system of linear algebraic equations of Eq. (21) to get the solution sequences $\Delta\{w\}_1^{(j)}$ and $\Delta\{\sigma\}_1^{(j)}$ ($j = 1, 2, 3, \ldots$) which are the increment parts of iterated values of $\{w\}_1$ and $\{\sigma\}_1$, respectively. In this iteration process, the iterating values of them will be replaced by the iterated values after they are got. The iteration is persistently carried out until some pre-specified conditions of precision criterion

$$\|\{w\}_1^{\{j\}} - \{w\}_1^{(j-1)}\| < \delta_1 \tag{22}$$

$$\|\{\sigma\}_1^{\{j\}} - \{\sigma\}_1^{(j-1)}\| < \delta_2 \tag{23}$$

are satisfied. Here, δ_1 and δ_2 are pre-given precision values. Then, we get the values of $\{w\}_1 = \{w\}_1^{(j)}$ and $\{\sigma\}_1 = \{\sigma\}_1^{(j)}$ for the case $\{R\}_1 = \{R\}_0 + \Delta\{R\}_1$.

Now, we replace the solutions $\{w\}_1$ and $\{\sigma\}_1$, and the load $\{R\}_1$ to $\{w\}_0$, $\{\sigma\}_0$, and $\{R\}_0$, respectively. And repeating the above calculations for a new load with load increment $\Delta\{R\}$, thus, we get a sequence of numerical solutions of them as long as the load increments are pre-chosen or appointed. It should be noted that we should evaluate which one element is either elastic or plastic in deformation in each step of calculation mentioned above, which consumes much more time of calculations except for the iteration for nonlinearity.

3.2. Finite element method for magnetic fields

From Section 2.3, we know that once the deflection of plate is known or given, one can get the distribution of magnetic fields. In this case, the boundary-value problem for magnetic fields, when the plate deflection is known, is linear. Hence, the solution of Eqs. (13)–(17) for the magnetic fields corresponds to that of the minimization of functional of magnetic energy (Zhou et al., 1995)

$$\Pi[\phi] = \frac{1}{2}\int_{\Omega^-(w)} \mu_0(\nabla\varphi^-)^2\,\mathrm{d}V + \int_{\Omega^+(w)} \mu_0\mu_\mathrm{r}(\nabla\varphi^+)^2\,\mathrm{d}V + \int_{S_0} \mathbf{n}_0\cdot\mathbf{B}_0\varphi^-\,\mathrm{d}s \tag{24}$$

Here, S_0 is a surface which encloses and is far away the ferromagnetic plate, while $\mathbf{n}_0$ is a unit vector outward normal to the surface S_0. Dividing the domains inside and outside the ferromagnetic plate, and applying to Eq. (24) similar to Zhou et al. (1995, 2000), we get a system of linear algebraic equations with matrix form

$$[K^{\mathbf{em}}(\{w\})]\{\Phi\} = \{B_0\} \tag{25}$$

where the unknown column $\{\Phi\}$ consists of those values of magnetic potential ϕ at the nodal points of elements; $\{B_0\}$ is the column of the magnetic fields related to those of applied magnetic field $\mathbf{B}_0$ at nodal points on the boundary surface S_0; and $[K^{\mathbf{em}}(\{w\})]$ is a global matrix of rigidity for the magnetic fields and is dependent on the deflection $\{w\}$. When the deflection $\{w\}$ and the applied magnetic field $\mathbf{B}_0$ are known, we can get the solution of magnetic potential by solving Eq. (25). Further, the magnetic field vector $\mathbf{H}$ may be obtained.

3.3. Iteration for coupling of deformation and magnetic fields

According to Eqs. (18) and (19), we find that the equivalent magnetic force in Eq. (18) is dependent upon the distribution of magnetic fields of Eq. (25) through the expression of magnetic force of Eq. (18), that is, $\{R\} = \{R(\{\Phi\})\}$, while the distribution of magnetic fields from Eq. (25) is wholly related to the deflection of plate $\{w\}$ which is determined by Eq. (19) related to the applied magnetic force $\{R\}$, or $\{\Phi\} = \{\Phi(\{w\})\}$. That is to say, the deflection of plate and the distribution of magnetic fields are nonlinearly coupled each other. In order to solve this coupling, we use the iteration like

$$[K_T(\{w\}_{1,n},\{\sigma\}_{1,n})]\{w\}_{1,n} = \{R(\{\Phi\}_{1,n})\} \tag{26}$$

$$[K^{\mathbf{em}}(\{w\}_{1,n})]\{\Phi\}_{1,n+1} = \{B_0\}_1 \tag{27}$$

in which $\{B_0\}_1$ is a column of pre-given applied magnetic field $\mathbf{B}_0$, and the subscript n $(= 1, 2, 3, \ldots)$ represents the iterative number.

4. Numerical results and discussions (case study)

In this section, we display some numerical results of buckling/snapping, bending and post-buckling/snapping for the elastic–plastic ferromagnetic plate with unmovable simple supports at two ends, and the effect of plastic deformation on these behaviors in a case study. The material and geometric parameters used in the case study are listed in Table 1. Using the numerical approach introduced in the previous section, firstly, we take a numerical test to evaluate the effectiveness of the numerical code for the case that the plate is only elastic. To this case, we get almost same results as those reported in Zhou et al. (2000) for the buckling, bending and post-buckling/snapping of elastic beam-plates with unmovable simple supports at the ends. Next, the numerical simulation of the characteristics of the ferromagnetic plate with magneto-elastic–plastic interaction is carried out.

Figs. 2 and 3 plot the characteristic curves of deflection of plate at position $x = 2L/5$ varying with the magnitude of applied magnetic field, B_0, with different incident angle α. In Fig. 2 exhibits the results of buckling/snapping and post-buckling/snapping phenomena for the plate of magneto-elastic–plastic inter-

Table 1
Material and geometrical parameters in the case study of numerical analysis

Young's modulus Y (Pa)	Poisson's ratio ν	Relative magnetic permeability μ_r	Yield stress σ_s (Pa)	Harden coefficient H' (N m)	Length L (m)	Thickness h (m)
2.1×10^{11}	0.3	1000	2.1×10^{8}	10000	1.00	0.01

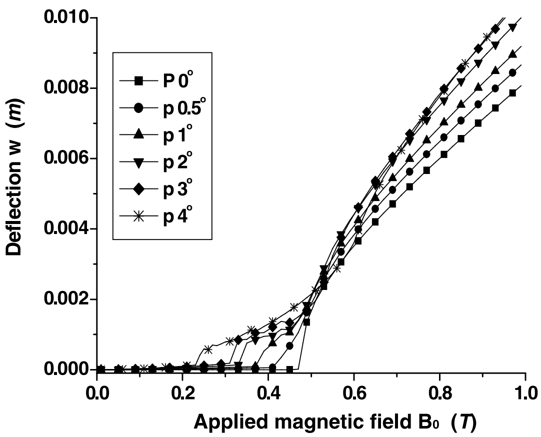

Fig. 2. Characteristic curves of deflection at $x = 2L/5$ versus the magnitude, B_0, of applied magnetic field with small incident angles $0° \leqslant \alpha \leqslant 4°$ to show the phenomena of buckling/snapping, bending, and post-buckling/snapping of the plate with magneto-elastic–plastic interaction. Here, "p1°" means that the plastic deformation is considered in the numerical code and the applied magnetic field has incident angle of 1°, and other notations in the legend have similar meaning.

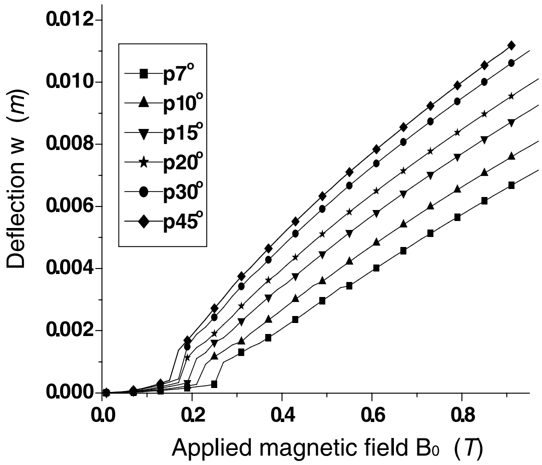

Fig. 3. Characteristic curves of deflection at $x = 2L/5$ versus the magnitude, B_0, of applied magnetic field with larger incident angles, $\alpha \geqslant 7°$. The notations in legend have the same meaning as in Fig. 2.

action when the incident angle is small, i.e., $\alpha \leqslant 4°$, whereas the path of bending deflection varying with the magnitude of applied magnetic field is shown in Fig. 3 when $\alpha \geqslant 7°$. When the plate is located in a transverse magnetic field ($\alpha = 0°$), it is found from Fig. 2 that the bending deflection suddenly generates from the trivial or zero deflection ($w(x) \equiv 0$) after the magnitude of magnetic field reaches to a critical value, which is the *buckling* phenomenon. When an oblique magnetic field is applied to the plate, Figs. 2 and 3 display that there is bending deformation before deflection increases rapidly as the magnitude of magnetic field is close to a critical value. After the magnitude is over the critical value with small increment, it is found that the plate deforms larger sharply when the incident angle is small, that is, the *snapping* phenomenon occurs, which corresponds to the jumping of deflection configuration from two semi-waves to single semi-wave (the results will be shown later as seen in Fig. 7). Due to that the geometric nonlinearity is taken into account in this numerical code, the path of deflection of post-buckling and post-snapping is tracked in the numerical simulation. From Figs. 2 and 3, we find that the level of jumping of the deflection changing is dependent upon the incident angle. When $\alpha = 0°$, the jumping of buckling instability is sharpest. The snapping phenomenon of the plate in oblique magnetic field occurs before plastic deflection is generated when $0° < \alpha \leqslant 1°$, while plastic deflection is generated in the plate, which corresponds to the first jumping displayed in the curves, before the snapping takes place in the interval $0° \leqslant \alpha \leqslant 4°$. When $\alpha \geqslant 7°$, we find from Fig. 3 that there is only the jumping caused by plastic deflection rather than snapping instability in the deflection path. The critical magnetic field when the plate undergoes plastic deformation, which is referred to as the yield magnetic field, decreases with increasing of the incident angle as shown in Fig. 4. Those denoted by "elastic region" and "elastic–plastic region" in Fig. 4, which are divided by the curve of dependence of the yield magnetic field on incident angle, imply that the plate structure, respectively, undergoes only elastic and/or elastic–plastic deformation when the parameters of applied magnetic field and incident angle are located in the corresponding region. Fig. 5 displays the evolution of plastic region(s) in the plate with the magnitude of applied magnetic field with incident angle $\alpha = 2°$ as an example. Fig. 5(b) plots two sub-regions of the plastic deflection in the plate when $B_0 = 0.37$ T, while Fig. 5(a) gives the plastic region(s) varying with the magnitude of applied magnetic field. To find the plastic region(s) from Fig. 5(a), one can draw a straight line paralleling to the horizontal axis of the figure to scale a magnitude of applied magnetic field. When the line intersects the curve in the figure, the plastic regions emerge with the end points corresponding to those intersect points on the closed curves which are constituted by the curve

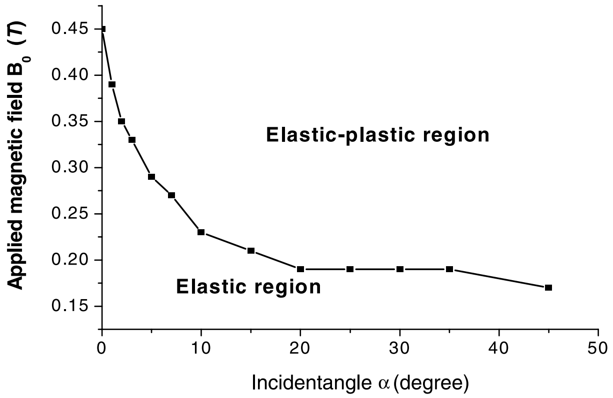

Fig. 4. The curve of yield magnetic field B_0^y varying with incident angle α when the plate just entering plastic deformation. "Elastic region" means that the plate generates only elastic deformation if the parameters (α, B_0) are located in the region, whereas "Elastic–plastic region" implies that the plate undergoes both elastic and plastic deformation when the parameters (or applied magnetic field) are in this region.

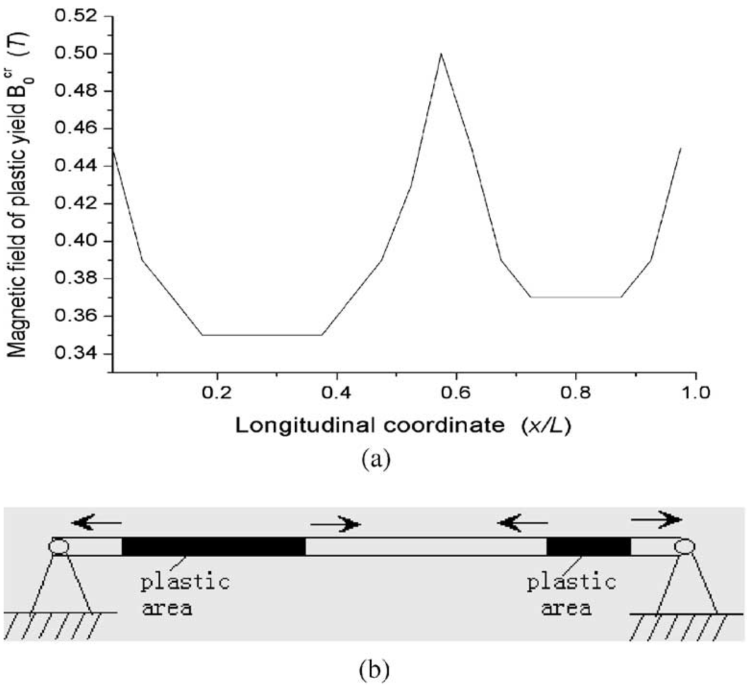

Fig. 5. A pattern of plastic area in the plate indicates where in the plate undergoes plastic deformation as the magnitude of applied magnetic field increases ($\alpha = 2°$): (a) plastic area varying with magnitude of the applied magnetic field; (b) $B_0 = 0.37$ T.

displayed in the figure and the drawn line. From the configuration of deflection, it is also found that there is anti-symmetry of deflection waveform when the plate is only in elastic deformation, whereas the anti-symmetry is broken down after the plate enters plastic deflection (see Fig. 5(b) shown an example of asymmetric plastic region and Fig. 7 plotted the asymmetric configuration of deflection curves).

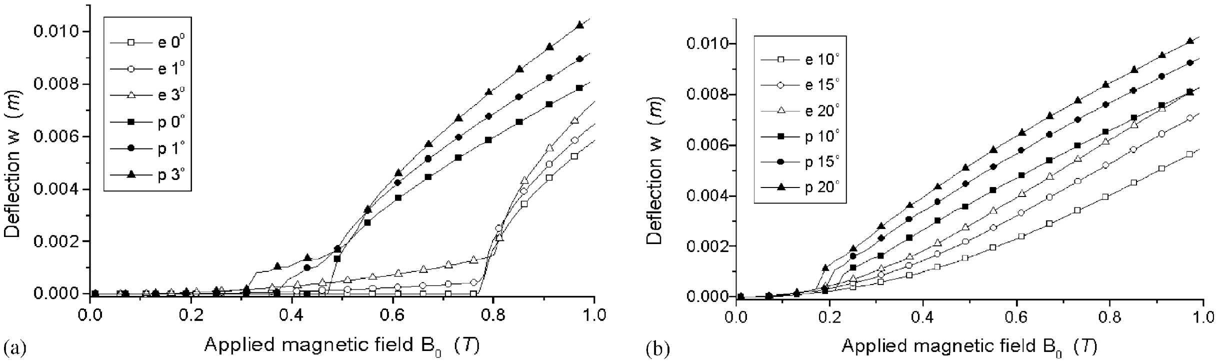

Fig. 6. Comparison of the deflection paths between the elastic plate and the elastic–plastic plate made of soft ferromagnetic materials under applied magnetic fields. Here, "e" and "p" in the figures imply that the plate is dealt with by elasticity and elastic-plasticity, respectively: (a) for small incident angle; (b) for larger incident angle.

Corresponding to the plate without consideration of plastic deflection, i.e., the magneto-elastic plate with geometrical nonlinearity, we display a comparison of the results obtained in this paper and those from the magneto-elastic plate with same material and geometrical parameters except for those related to plasticity, which is plotted in Fig. 6. In Fig. 6, the symbols of "e" and "p" imply those results of the magneto-elastic plate and the magneto-elastic–plastic plate, respectively. It is shown that the critical values of applied magnetic field of the magneto-instability for the elastic–plastic plate are smaller than those for the elastic plate when the plates are located in the magnetic field with same incident angle. The deviation of critical values between them increases with decreasing of the incident angle. This result tell us that the stability of the plate is sensitive to the plastic deformation when incident angle is not large enough, which is most important to the design of safety of the structure. As same as those given in Zhou et al. (2000) for the deflection configuration of the plate with elastic deformation only, Fig. 7 exhibits an evolution of configuration of the plate with elastic–plastic deflection when the plate is in an oblique magnetic field with small angle ($\alpha = 2°$). It is known from the figure that the deflection configuration of the plate jumps from two

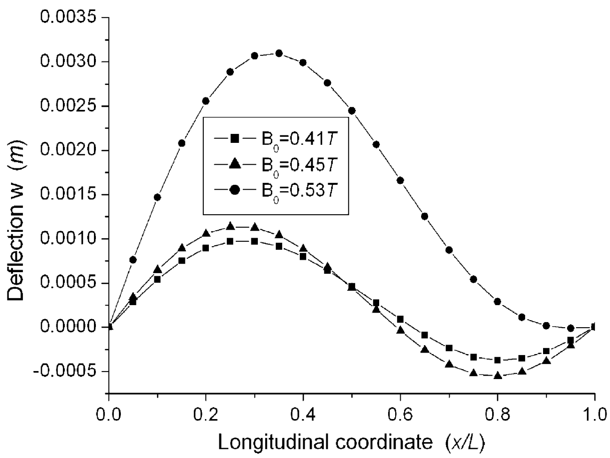

Fig. 7. Changing of configuration of deflection of the plate in an oblique magnetic field with the magnitude of the magnetic field ($\alpha = 2°$).

semi-waves to a single semi-wave as the magnitude of applied magnetic field increases across the critical value of the magneto-instability, which is similar to the phenomenon in Zhou et al. (2000). The numerical results show that when $\alpha \leqslant 5°$, this jumping phenomenon from two semi-wave deflection configuration into a single semi-wave one occurs, while does not when $\alpha \geqslant 7°$. Since this jumping leads to a rapid increase of deflection of the plate at position $x = 2L/5$, thus, there is a sudden drop of the deflection as seen in Figs. 2 and 3 for α somewhere between 4° and 7° when the applied magnetic fields is large enough (about $B_0 > 0.7$ T).

5. Conclusions and remarks

A numerical code for the magneto-elastic–plastic interaction of ferromagnetic beam-plates with geometrical nonlinearity is established and some numerical results for a case study of the buckling/snapping, bending, post-buckling and post-snapping behaviors to the soft ferromagnetic beam-plate with geometrical nonlinearity and unmovable simple supports at two ends are displayed in this paper. When the incident angle of applied magnetic fields is in the region of $0° \leqslant \alpha \leqslant 1°$, the plastic deflection is generated only after the plate loses its stability, while the plate undergoes plastic deflection before the plate snaps when $\alpha \geqslant 2°$. It is found that the critical magnetic field for the magneto-stability, either buckling when $\alpha = 0°$ or snapping if $\alpha \neq 0°$, of the plate structure is sensitive to the plastic deformation that makes the critical value lower comparing to the corresponding elastic plate. As the incident angle α increases, the yield magnetic field at which the plate generates plastic deformation decreases.

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